CHAPTER SIX

SIMULTANEOUS EQUATIONS

* In simultaneous equation, one may be given two equations, containing two unknown variables.
* To solve these equations simultaneously means that you must determine a set of values for these unknown variables, such that when these values are substituted into any of the two equations in turn, each will be satisfied.
* Different methods such as the elimination method, the substitution method or the graphical method can be applied.

(Q1) Solve the equations given simultaneously

a + b = 10

a - b = 4

N/B

(1) Let the first equation be equation (1) and the second one be equation (2).
(2) Ensure that the second letters or the unknown variables of each of the equations (i.e. the b in 'this case) are of the same value.
(3) Ensure also that one of the signs is positive while the other is negative.
(4) When all these conditions have been satisfied, the two equations are added up.

Solution

a + b = 10----equation (1)
a - b = 4------equation (2)

Since each b has the same value as the other one , and we have both the positive as well as the negative signs being available, we add them together.

i.e.

a + b =10 ----------equation (1)

+a - b = 4 -----------equation (2)

2a = 14

$\Rightarrow$ 2a = 14 $\Rightarrow$ a = $\frac{14}{2}$ =7

N/B: When positive b is added to negative b, we get 0 for which there is no need to indicate.

In order to find the value of b, substitute or put a = 7 into either equation (1) or equation (2).

Substituting a =7 into eqn. (1)

$\Rightarrow$ a + b = 10

$\therefore$ 7 + b = 10

$\Rightarrow$ b = 10 - 7 = 3 $\Rightarrow$ b = 3.

N/B : The values a = 7 and b = 3 when substituted into either equation (1) or equation (2) must satisfy or balance it.

i.e. a + b = 10 ----------eqn (1)

$\Rightarrow$ 7 + 3 = 10

$\Rightarrow$ 10 = 10.

Also a - b = 4-------------eqn (2)

$\Rightarrow$ 7 - 3 = 4

$\Rightarrow$ 4 = 4.

(Q2) Solve the following equations simultaneously

$x + y = 3$ and $x - y = -1$.

<u>Soln</u>

Let $x + y = 3$ …………….eqn (1)

And $x - y = -1$………… eqn (2)

Adding the two equations up

$\Rightarrow x + y = 3$

$+ \underline{x - y = -1}$

$\underline{\ \ 2x \qquad =2}$

$\therefore 2x = 2 \Rightarrow x = 2/_2 = 1.$
Substitute $x = 1$ into eqn (1) to find the value of y
i.e. $x + y = 3 \Rightarrow 1 + y = 3,$
$\Rightarrow y = 3 - 1 \Rightarrow y = 2.$

The values of x and y which satisfy simultaneously the two given equations are

$x = 1$ and $y = 2.$

N/B: The above method used is referred to as the elimination method.

The same question could have been solved, using the substitution method, which is illustrated next:

$x + y = 3$ …………….eqn (1)

$x - y = -1$………… eqn (2)

From eqn (1) , $x + y = 3$

$\Rightarrow x = 3 - y$. Substitute $x = 3 - y$ into eqn (2)

i.e $x - y = -1 \Rightarrow$

$(3 - y) - y = -1, \Rightarrow 3 - y - y = -1,$

$\therefore 3 - 2y = -1 \Rightarrow -2y = -1 - 3,$

$\Rightarrow -2y = -4 \Rightarrow \frac{-2y}{-2} = \frac{-4}{-2},$

$\Rightarrow y = 2.$

Substitute $y = 2$ into eqn (1) to find x , i.e. $x +\ \ y = 3\ \Rightarrow x + 2 = 3,$

$\Rightarrow x = 3 - 2$

$\Rightarrow x = 1$.

(Q3) Solve the following equations simultaneously:
$p + 2q = 12$
$p - q = 3$

Soln

Let $p + 2q = 12$ …………………eqn (1) and $p - q = 3$ … … … … …eqn (2)

N/B: Considering these two equations, the values of q are not the same, or equal.

– In order to make them equal, 2 is used to multiply through eqn. 2 (i.e $p - q = 3$)

– Multiplying through eqn (2) by 2

$\Rightarrow 2 \times p - 2 \times q = 2 \times 3$

$\therefore 2p - 2q = 6$ …………………eqn (3) –
After multiplying through an equation with any number, it changes into another equation

–For this reason, eqn (2) changes into eqn (3) after using 2 to multiply through it.

– We now consider equation (1) and equation (3)

i.e.

$p + 2q = 12$ …………………eqn (1)

$2p - 2q = 6$ … … … … … … … …..eqn (3)

Since each q has the same value as the other one, with both the negative and positive signs being present, we add them up.

i.e

$$
\begin{aligned}
&.p + 2q = 12 \\
+ \quad &2p - 2q = 6
\end{aligned}
$$

3p = 18

$3p = 18 \Rightarrow p = \frac{18}{3} = 6$.

Substitute $p = 6$ into eqn (2)

i.e $p - q = 3, \Rightarrow 6 - q = 3,$

$\therefore 6 - 3 = q \Rightarrow q = 3$.

The required answer is p = 6 and q = 3.

Method 2 (Substitution Method):

$p + 2q = 12$……………….eqn (1)

$p - q = 3$ …………………eqn (2)

From eqn (2) $p - q = 3 \Rightarrow p = 3 + q$.

Substitute $p = 3 + q$ into eqn (1) i.e $p + 2q = 12$

$\Rightarrow (3 + q) + 2q = 12,$

$\Rightarrow 3 + q + 2q = 12$

$\Rightarrow 3 + 3q = 12, \Rightarrow 3q = 12 - 3$

$\Rightarrow 3q = 9, \Rightarrow q = \frac{9}{3} = 3$

$\therefore$ q = 3.

Now substitute q = 3 into eqn (1) or eqn (2) to find p.

Using eqn (1) i.e $p + 2q = 12$

$\Rightarrow p + 2(3) = 12 \Rightarrow p + 6 = 12,$

$\Rightarrow p = 12 - 6 \Rightarrow p = 6.$